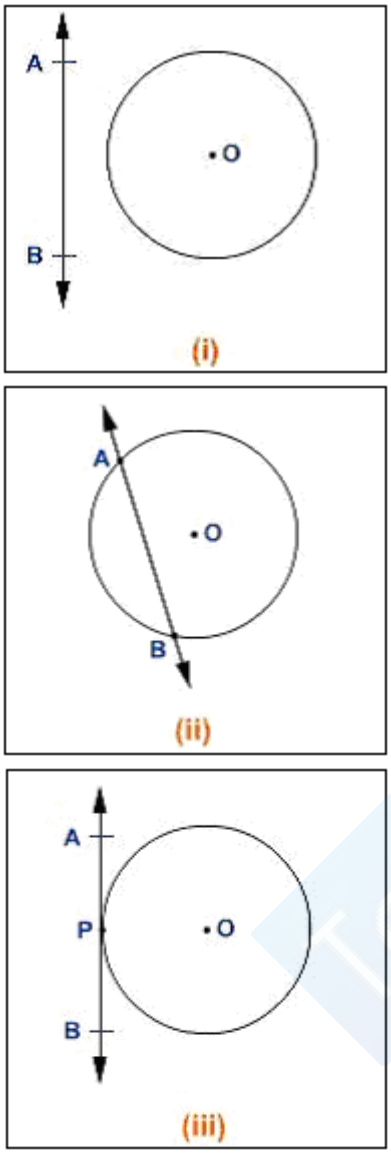
Circles

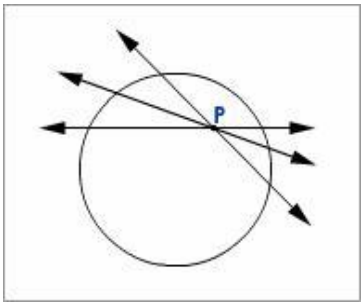
Tangent to a Circle

A tangent is a line touching a circle at one point

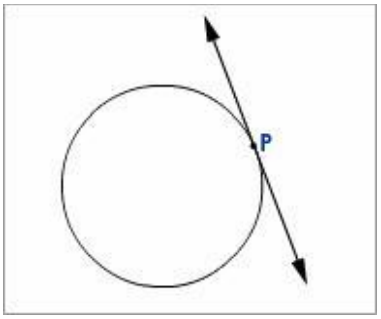


1. **Non-intersecting line** - fig (i): The circle and the line AB have no common point.
2. **Secant** - fig (ii): The line AB intersects the circle at two points A and B. AB is the secant of the circle.
3. **Tangent** - fig (iii): The line AB touches the circle at only one point. P is the point on the line and on the circle. P is called the point of contact. AB is the tangent to the circle at P.

Number of Tangents from a Point on a Circle

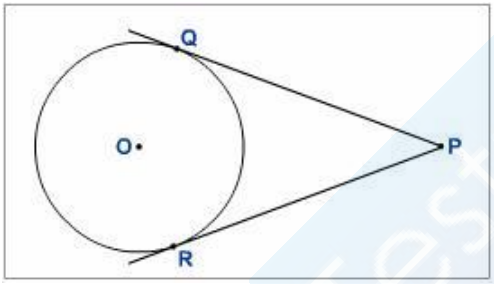


From a point inside a circle, no tangents can be drawn to the circle.



From a point on a circle, only 1 tangent can be drawn to the circle.

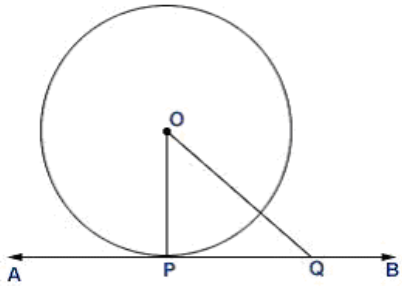
In this figure, P is a point on the circle. There is only 1 tangent at P. P is called the point of contact.



From a point outside a circle, exactly 2 tangents can be drawn to the circle. In this figure, P is the external point. PQ and PR are the tangents to the circle at points Q and R respectively. The length of a tangent is the length of the segment of the tangent from the external point to the point of contact. In this figure, PQ and PR are the lengths of the 2 tangents.

Theorem 1:

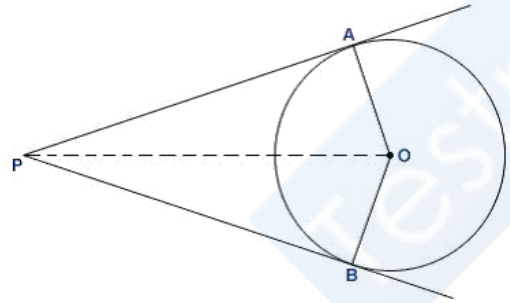
The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given:
AB is a tangent to the circle with centre O. P is the point of contact. OP is the radius of the circle.
To prove:
OP $\perp$ AB
Proof:
Let Q be any point (other than P) on the tangent AB.
Then Q lies outside the circle.

For any point Q on the tangent other than P.
$\Rightarrow$ OP is the shortest distance between the point O and the line AB.
$\Rightarrow$ OP $\perp$ AB
($\because$ The shortest line segment drawn from a point to a given line, is perpendicular to the line)
Thus, the theorem is proved.
From the above theorem,
1. The perpendicular drawn from the centre to the tangent of a circle passes through the point of contact.
2. If OP is a radius of a circle with centre O, a perpendicular drawn on OP at P, is the tangent to the circle at P.

Theorem2:
The lengths of tangents drawn from an external point to a circle are equal.



Given:
P is an external point to a circle with centre O. PA and PB are the tangents from P to the circle. A and B are the points of contact.
To prove:
PA = PB
Construction:
Join OA, OB, OP.
Proof:
In triangle APO and BPO,

Statement	Reason
OA = OB	Radii of the same circle
	The radius is perpendicular to the tangent at the point of the contact.
OP = OP	Common
	By SAS postulate
PA = PB	CPCT(Third side of the triangles)

From the above theorem,

1. (CPCT) This states that the two tangents subtend equal angles at the centre of the circle
2. PO (CPCT) The tangents are equally inclined to the line joining the point and the centre of the circle.
   Or the centre of the circle lies on the angle bisector of the $\angle APB$.